Distributional deep Q-learning with CVaR regression

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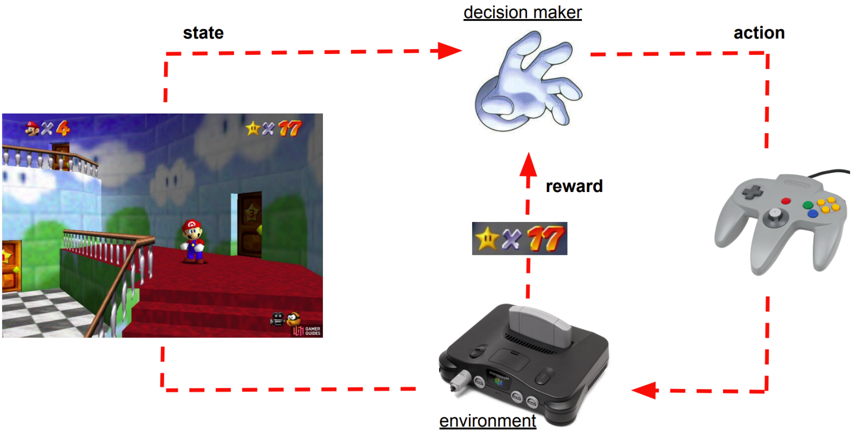
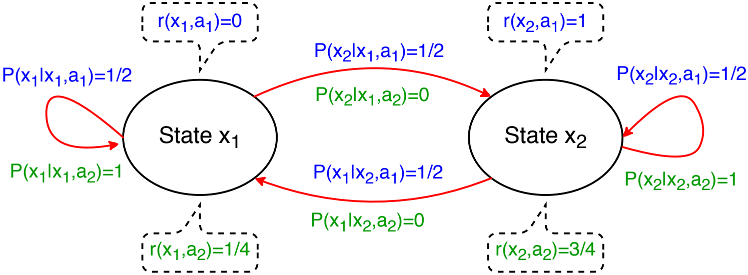
Context: Sequential decision-making



Markov decision process (MDP)

An MDP [Puterman, 2014] is characterized by: states x, actions a, rewards $r(x, a, x')$ and transition probabilities $P(x'|x, a)$.



The control task

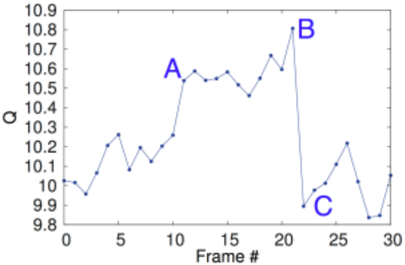
Optimality. Find a strategy π (mapping any state x to an action $\pi(x)$) that is optimal in terms of *expected* cumulative discounted return (for some discount factor $0 \leq \gamma < 1$):

$$Q^*(x,a) = \max_{\pi} \; Q^{\pi}(x,a) := \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r(X_t, A_t, X_{t+1}) \;\middle|\; X_0 = x, A_0 = a\right]$$

with states $X_{t+1} \sim P(\cdot|X_t, A_t)$ and actions $A_{t+1} = \pi(X_{t+1})$.
Reinforcement learning (RL). Learn an optimal strategy without knowing the transitions probabilities $P(x'|x,a)$ or the reward function: an RL agent only observes empirical transitions (x_t, a_t, r_t, x_{t+1}).

Deep Q-Network (DQN)

The DQN agent [Mnih et al., 2013] learns Q^* with a deep neural net Q_θ with parameters θ: successfully plays Atari games!

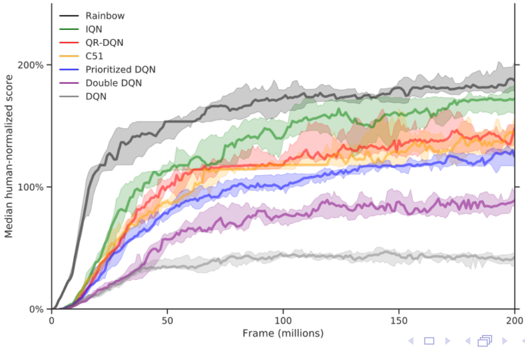


Distributional RL [Bellemare et al., 2017]

In distributional RL, the agent learns the whole probability distribution of the total return:

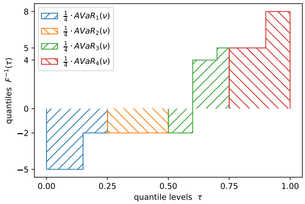
$$\text{Law}\left(\sum_{t\geq 0}\gamma^t r(X_t, A_t, X_{t+1}) \,\middle|\, X_0 = x, A_0 = a; \pi\right).$$

In contrast, RL only focuses on the expected value $Q^\pi(x,a)$ of this distribution. On Atari games, distributional RL outperforms RL!



Our approach: Wasserstein-2 projection

We use the W_2-projection to approximate distributions by a fixed number of values called “AVaRs” [Achab and Neu, 2021].



Algorithm 3 Discrete AVaR computation

Input: $N \geq 1$ and discrete distribution $\nu = \sum_{j=1}^{M} p_j \delta_{v_j}$ with $M \geq 1$.

Sort atoms:

$$v_{\sigma(1)} \leq \cdots \leq v_{\sigma(M)} \quad \text{with } \sigma \quad \text{an argsort permutation}$$

Reorder probability-atom pairs:

$$(p_j, v_j) \leftarrow (p_{\sigma(j)}, v_{\sigma(j)})$$

Compute AVaRs:

$$\text{AVaR}_i(\nu) = N \cdot \sum_{j=1}^{M} \left[\min\left(\frac{i}{N}, \sum_{j' \leq j} p_{j'} \right) - \max\left(\frac{i-1}{N}, \sum_{j' \leq j-1} p_{j'} \right) \right]_+ \cdot v_j$$

Output: $\text{AVaR}_1(\nu), \ldots, \text{AVaR}_N(\nu)$.

Distributional DQN with AVaRs

We propose two new deep and distributional RL algorithms based on AVaR targets.

Algorithm 2 SAD-DQN update

Input: $(Q_{1;\theta}, \dots, Q_{N;\theta})$ with deep Q-net parameters θ, target network θ^-, transition $(x, a, r(x, a, x'), x')$, mixing ratio $\alpha \in (0, 1)$ and learning rate $\eta > 0$.

Target state-action value function:

$$Q_{\theta^-} \leftarrow \frac{1}{N} \sum_{i=1}^{N} Q_{i;\theta^-}$$

Target atomic distribution in (x, a):

$$\mu_{\theta^-}^{(x,a)} \leftarrow \frac{1}{N} \sum_{i=1}^{N} \delta_{Q_{i;\theta^-}(x,a)}$$

Mixture update:

$$\nu \leftarrow (1-\alpha)\mu_{\theta^-}^{(x,a)} + \alpha \delta_{r(x,a,x') + \gamma \max_{a'} Q_{\theta^-}(x', a')}$$

Perform a gradient descent step w.r.t. θ on the squared Wasserstein-2 loss function:

$$\theta \leftarrow \theta - \eta \nabla_\theta \frac{1}{N} \sum_{i=1}^{N} \left(Q_{i;\theta}(x,a) - \mathrm{AVaR}_i(\nu)\right)^2$$

Output: Updated parameters θ.

Algorithm 3 MAD-DQN update

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Mixture update:

$$\nu \leftarrow (1-\alpha)\mu_{\theta^-}^{(x,a)} + \frac{\alpha}{N} \sum_{i=1}^{N} \delta_{r(x,a,x') + \gamma Q_{i;\theta^-}(x', a^*)},$$

where $a^* \leftarrow \arg\max_{a'} Q_{\theta^-}(x', a')$

Perform a gradient descent step w.r.t. θ on the squared Wasserstein-2 loss function:

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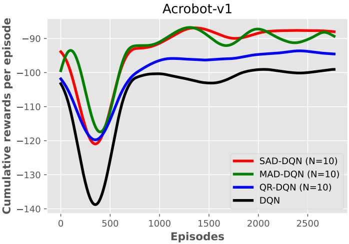
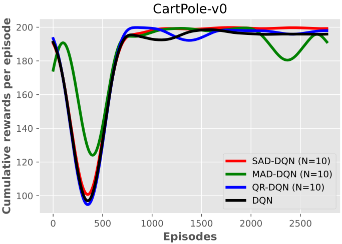
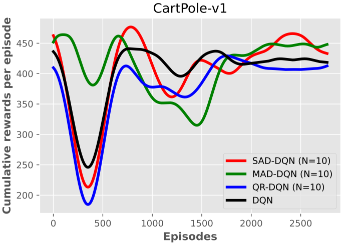
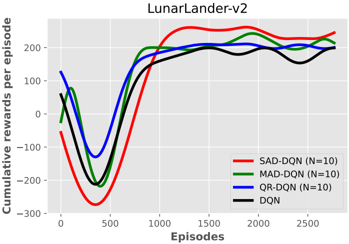
Experiments - OpenAI Gym

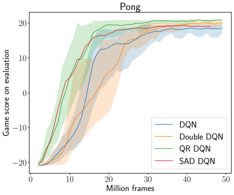




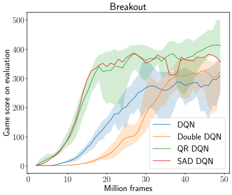




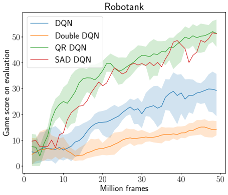
Experiments - Atari games



(a)



(b)



(c)

Figure: Performance on three Atari games.

References

Achab, M. and Neu, G. (2021).
Robustness and risk management via distributional dynamic programming.
arXiv preprint arXiv:2112.15430.

Bellemare, M. G., Dabney, W., and Munos, R. (2017).
A distributional perspective on reinforcement learning.
In *International Conference on Machine Learning*, pages 449–458. PMLR.

Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., and Riedmiller, M. (2013).
Playing atari with deep reinforcement learning.
arXiv preprint arXiv:1312.5602.

Puterman, M. L. (2014).
Markov decision processes: discrete stochastic dynamic programming.
John Wiley & Sons.